Analytic Mechanics: Discussion Worksheet 4

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This week, we're going to work on some very basic Lagrangian mechanics problems, with an eye toward slowing down (compared to the torrid pace of the homeworks) and trying to understand every step of the process.

1 Ruminations on the free particle

1.1 Write down the Lagrangian for a free particle moving in two dimensions, x and y.

1.2 Use this Lagrangian to calculate the canonical momenta that are conjugate to x and y, p_x and p_y respectively.

1.3 Suppose that you added a potential $U(x, y)$ to your system. Write out the Euler-Lagrange equations, using p_x instead of $\frac{\partial \mathcal{L}}{\partial \dot{x}}$ (and similarly for y). Do you recognize these equations from 7A?

1.4 Forgetting about $U(x, y)$ now, assume that the particle is compelled to move in a circular path of diameter 1, centered at the origin.

1.4.1 Write the equation of constraint.

1.4.2 Identify g_x and g_y for use with Lagrange multipliers.

1.4.3 Write out the full Euler-Lagrange equations, with undetermined multipliers.

1.4.4 Differentiate the equation of constraint twice, and solve for the acceleration in the x-direction.

1.4.5 Use what you've written so far, along with conservation of energy, to solve for the force of constraint required to keep the particle in a circular trajectory. Having found this, can you write it in a nice vectorized form?

1.4.6 [optional] The forces of constraint can be integrated to find an effective Lagrangian that would have yielded the solution to the constrained problem ***without*** any constraints. Try to write down such an effective Lagrangian, either explicitly or in integral notation.

1.4.7 [optional] In the previous section, can you write the Lagrangian in terms of coordinates such that one is cyclic? (Think; it's a trick question.) If so, what is the cyclic coordinate? If not, why not?

2 Particle in a gravitational field

2.1 Again in two coordinates, write the Lagrangian for a particle in a uniform gravitational field $U = mgy$. Solve for the motion of the particle.

2.2 Suppose that the particle is constrained to a parabolic surface, $y = -x^2$.

2.2.1 Write the equation of constraint, and find g_x and g_y. Are these choices of g's unique? Why [not]?

2.2.2 Write out the full (including multipliers) Euler-Lagrange equations.

2.2.3 Use the E-L equations with the equation of constraint to find an equation of motion for the system.

2.2.4 Find the point at which the force on the parabolic constraint changes sign (direction).